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COMPOSED GRAPH REWRITING (ILLUSTRATED IN GROOVE)

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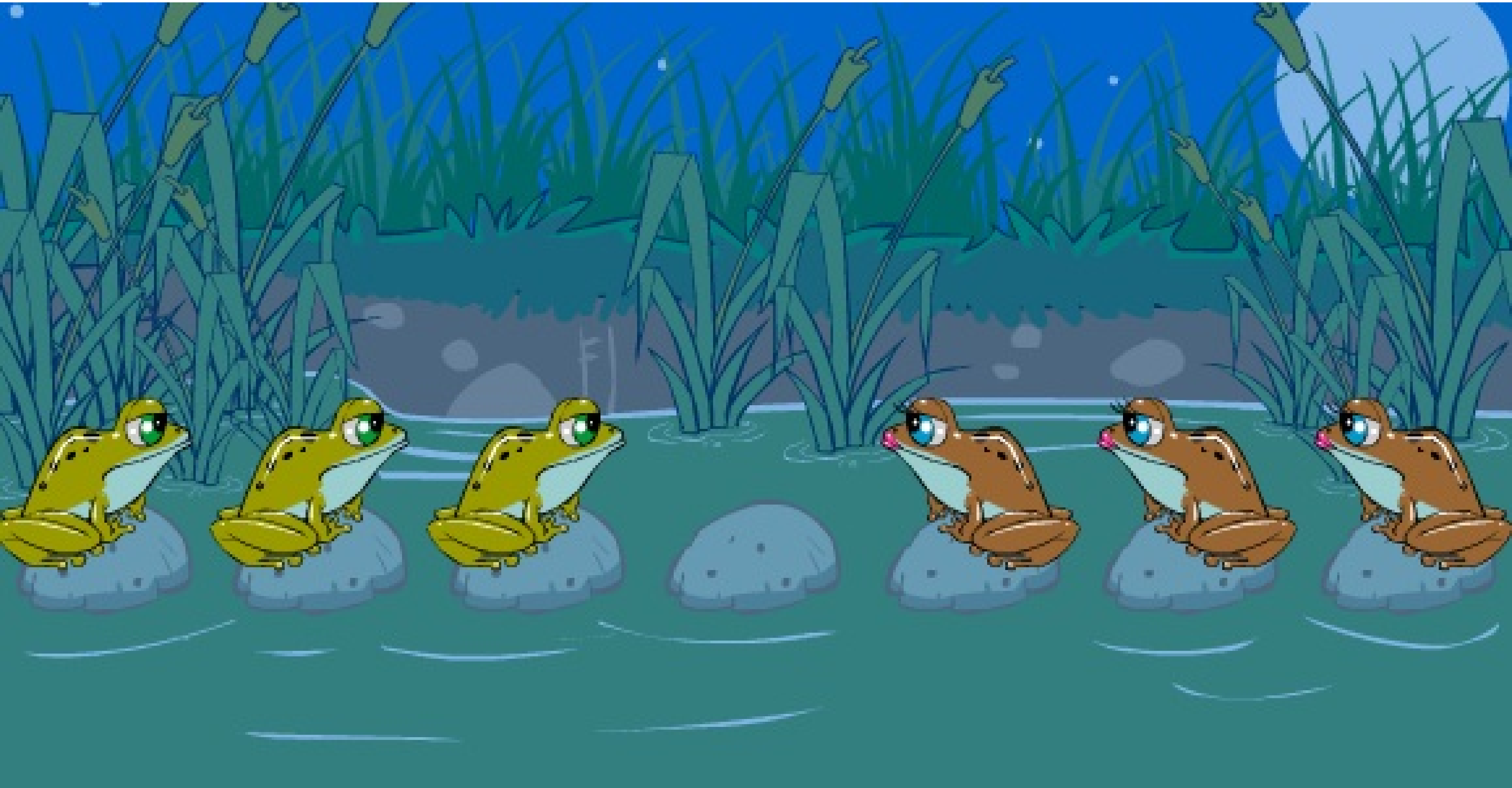
~~GRAPH~~ REWRITING FRAMEWORK

- Main ingredients
  - Graphs $G \in Graph$: the objects being rewritten
    - (Partial) morphisms $f \in Morph \subseteq Graph \times Graph$
  - Rules $r \in Rule$: embodiment of types of change to (certain) graphs
  - Matches $m \in Match$: places in graph where rule can be applied
- Matching function $M: Rule \times Graph \rightarrow 2^{Match}$
  - $M^r(G)$ denotes the set of matches of r in G
- Rule application $A: Graph \times Rule \times Match \rightharpoonup Morph \times Graph$
  - $G \Rightarrow^{r,m,f} H$ denotes $A(r, m) = (f, H)$
  - Match resolves non-determinism: A is a (partial) *function*
  - Defined on (G, m, r) if and only if $m \in M^r(G)$
  - Match m and morphism f often omitted: $G \Rightarrow^{r,m} H$ or $G \Rightarrow^{r} H$
- Nothing in the above is specific to graphs
  - Other rewriting formalisms: strings, terms, proofs, bigraphs, ...

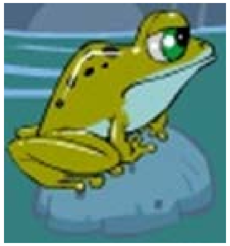
EXAMPLE: FROG PUZZLE



Demo using GROOVE: http://sf.net/projects/groove

GRAPH TRANSITION SYSTEMS

- Graph transition system (GTS): tuple $S = \langle Q, R, \rightarrow, \iota \rangle$
  - States $q \in Q$, each with associated graph G_q
  - Rules $R \subseteq Rule$
  - Transition relation $\rightarrow \subseteq Q \times R \times Match \times Morph \times Q$
    - $q \rightarrow^{r,m,f} q'$ only if $G_q \Rightarrow^{r,m,f} G_{q'}$ (not necessarily *if*!)
    - Again, we may omit m and (more often) f
  - Initial state $\iota \in Q$



- Frequently: uncontrolled (unscheduled) GTS
  - $Q \subseteq Graph$ and $G_q = q$
  - Every transformation generates a (unique) transition
    - Here, $q \rightarrow^{r,mf} q'$ whenever $G_q \Rightarrow^{r,m,f} G_{q'}$
  - S is completely determined by $\langle R, \iota \rangle$

GRAPH GRAMMARS

- Rule system R with initial graph defines *graph language*
  - Language of a GTS = set of graphs of reachable terminal states
  - For instance: language of trees, 2-coloured graphs, flow graphs
  - Generalises string grammars
- Common technique: every rule consumes a “non-terminal”
  - When all non-terminals are consumed, state is terminal
  - Context-freedom: LHS is *only* a single non-terminal
- Transition systems typically uncontrolled and infinite

GRAPH PRODUCTION SYSTEMS

- Rule system R defines relation ρ_R over graphs
  - $(G, H) \in \rho_R$ iff H is the graph of a reachable state when $G_\iota = G$
  - H is "produced" from G
- Often, ρ_R is meant to be a (partial or total) function
  - Only (or at most) one reachable terminal state for any start graph
  - Which can be found (or its absence confirmed) quickly and reliably
- Transition systems typically finite
  - Infinite paths are very undesirable
  - Schedules can help to find short paths ("evaluation strategies")
- Examples
  - Normal form computations
    - E.g., functional programming, theorem proving
  - Model transformation
    - E.g., "construct the flow graph from an abstract syntax graph"

GRAPH-BASED BEHAVIOURAL SEMANTICS

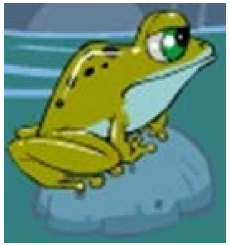
- Graph transition system describes evolution of system
  - Either trace set or full transition system is relevant
  - Often, reachable terminal state = deadlock = error
- Transition systems
  - Typically contain cycles
  - Typically are non-deterministic
  - May very well be infinite (though this is often an error)
- Control is often very useful

OUTLINE OF THIS TUTORIAL

- Framework for (graph) transformation
  - Rule+match+tracing morhphism-labelled transition systems
  - Usage scenarios: grammars, production systems, semantics
- Composition mechanisms: when simple rules are not enough
  - Amalgamation
    - Multi-nodes
    - Nested rules
  - Parameters
    - Input, output
  - Supervisory control
    - Programmed graph transformation
    - Atomicity
    - Transformation units
  - Strategic control

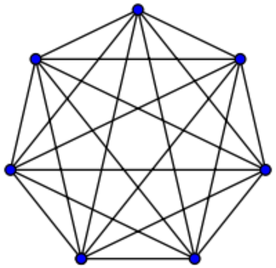
AMALGAMATION

- Simple rules are limited
  - Effect is local and bounded / rules not generic
  - Example: rewrite (maximal) complete subgraph to star graph
  - *Note*: limitations can be advantageous!
- Idea: apply one or several rules simultaneously
- Formal interpretation
  - Take multiple matches of one or more rules (in the same graph)
  - Duplicate the rules per match and take their union
  - Apply the composed rule
  - Amalgamated rules may be nested, so *union* ≠ *disjoint union*
- This is *not* always the same as *repeatedly* applying rules
  - All composed rules are applied to the same graph
  - Conflicts are resolved (or prevent rule application)
  - Matches cannot appear or disappear



GENERALISATION: FAMILIES OF RULES

1. Through amalgamation
   - Copying/gluing subrules arbitrary number of times
2. As the language of a grammar over rules
   - As seen yesterday in Vladimir Zamdzhiev's presentation



- I feel the latter is probably strictly more expressive
  - At least to express transformation in 1 rule
- There are other well-known cases where amalgamation fails
  - Matching/processing all elements of a list
  - Copying a graph of arbitrary structure
- Copied subrules cannot refer to one another
  - Context-free in some sense?
  - Requires second-order logic

SUPERVISORY CONTROL

- Explicitly determine the order of rule application
  - Programmed graph transformation
- Typical constructs
  - Try a rule, do something else if rule is not applicable
  - Do rules in sequence
  - As long as possible apply a rule/set of rules

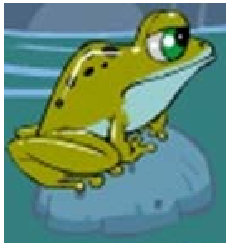
RULE PARAMETERS

- Output parameters
  - Expose part of the match on the label
  - Primarily for observation
- Input parameters
  - Partially determine the match
  - Primarily for control
  - Pragmatic reasons: to avoid “guessing” attribute values

Issue

  - Node type parameters expose node identities
  - Supposed to be internal/unknowable

TRANSACTIONS



- If next rule in a sequence fails, state is terminal
  - This may not be the intended meaning
- Transaction implies:
  - All-or-nothing behaviour
  - Backtrack & abandon path if it leads to terminal state
  - Abandoned part is *not* in the GTS!
- Implicit in the semantics of try/else and alap
  - Body of alap should "fail" on terminal states
  - Not just if first rule is inapplicable

TRANSFORMATION UNITS

- Named control abstractions
  - Signature consisting of (input and output) parameters
  - Control program as body
- Behave as (composed) rules
  - Single transition in GTS
  - Labelled by unit name & tracing morphism
  - Body is executed as transaction (= atomically)
- Groove: Recipes
  - Example: frogs
  - Freak example: fibonacci

STRATEGIC CONTROL

- Often, one does not want to explore entire transition system
  - State space is too large
  - State space known to be confluent
- Exploration strategies
  - Simulation mode
    - Linear exploration
  - Search mode, e.g. for property violations (LTL, invariant)
    - Depth-first rather than breadth-first
  - Optimisation mode: find “good” solution
    - Local rather than global optimum
- Heuristics
  - Decide which path to explore first
  - Problem-dependent vs. problem-independent
- *Supervisory control restricts LTS, strategic control does not!*

EVALUATION

- Why are simple rules not enough?
  - Effect only local, not generic
  - Require to put control elements into graphs
  - Granularity not appropriate for problem at hand
  - Monolithic, no reuse of common elements
- Composition mechanisms
  1. In space: families of rules
  2. In time: supervisory control, transformation units
- Disadvantages
  1. More complex rules: *reasoning becomes harder*
  2. Loss of declarative nature: *reasoning becomes harder*
- This is a fake objection!
  - Systems that benefit from composition mechanisms *are* complex
  - Composition partially relieves this, partially shifts it elsewhere